Thermohaline Convection in a Porous Medium in the Presence of Magnetic Field and Rotation

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Abstract

The onset of Benard convection in a horizontal porous layer permeated by an incompressible, thermally and electrically conducting solute fluid under the effect of a uniform vertical magnetic field and a uniform vertical rotation is investigated. The porous layer is assumed to be governed by the Brinkman model. Analytical and numerical solutions are obtained for the cases of stationary convection and overstability. The critical thermal Rayleigh numbers are obtained for different values of the permeability of porous medium, Chandrasekhar number and Taylor number for different cases of solute boundary conditions. The related eigenvalue problem is solved using the Chebyshev polynomial Tau method.

Keywords

Thermohaline; Thermal Instability; Porous Medium; Magnetic Field; Rotation

Introduction

Thermal instability theory has attracted considerable interest and has been recognized as a problem of fundamental importance in many fields of fluid dynamics. This theory has been enlarged by the interest in hydrodynamic flows of electrically conducting fluids in the presence of magnetic field and rotation (Chandrasekhar 1960).

The onset of convection induced by thermal and solute concentration gradients in a horizontal layer of a viscous fluid is studied by Nield (1967) when the layer is heated from below and the solute concentration increases upwards for general set of boundary conditions. Nield (1968) studied the thermohaline convection in a porous medium for a general set of boundary conditions when the fluid is heated and soluted from below. The thermosolutal convection in a fluid layers heated and soluted from below in a porous medium has been studied by Sharma and Kumari (1992) to include, separately, the effects of a uniform horizontal magnetic field and a uniform rotation when the boundaries are free. Khare and Sahai (1992) studied the thermosolutal convection in a heterogeneous fluid layer heated and soluted from below in porous medium when the boundaries are free using Brinkman – Oberbeck – Boussinesq equations for the cases of stationary and overstability convection. This problem has been extended by Khare and Sahai (1993) to include the effect of magnetic field and by Khare and Sahai (1994) to include the combined effects of rotation and magnetic field. The thermosolutal convection of a horizontal layer of an incompressible viscous fluid in the presence of vertical magnetic field is discussed by Al-Aidrous and Abdullah (2001). Related problems on the thermosolutal convection in a porous medium have been discussed by Sunil et al. (2005), Singh and Kumar (2011), Nield and Bejan (2013) and others.

This work studies convective instability in a horizontal porous layer permeated by an incompressible, thermally and electrically conducting solute fluid using Brinkman model in the presence of a uniform vertical magnetic field and a uniform vertical rotation. Analytical solutions were obtained when both boundaries are free and numerical results were presented for the cases of free and rigid boundaries. The numerical computations were performed using the Chebyshev Tau method.

The implementation of Chebyshev Tau method is highly useful in obtaining accurate eigenvalues for one

layer and two layers problems. The application of a D^2 Chebyshev tau methods to a variety of subject areas is discussed by Abdullah and Lindsay (1990, 1991a, 1991b), Lindsay and Ogden (1992), Abdullah (2000), Straughan (2001, 2002), Al-Aidrous and Abdullah (2005), Gheorghiu (2007), Gheorghiu and Dragomirescu (2009), Banjar and Abdullah (2010) and others.

Mathematical Formulation

Consider an infinite horizontal layer of porous medium confined between two parallel horizontal boundaries and permeated by an incompressible thermally and electrically conducting viscous solute fluid. The porous layer is subjected to a constant vertical magnetic field with a magnetic field intensity ***B*** *(=(0,0,B))* and to a rotation about the x_3 axis with angular velocity Ω. Gravity g acts in the negative direction of x_3.

Let θ be the absolute temperature of the fluid C the solute mass concentration, T_0, $T_0-\tilde{T}$, $\tilde{T}>0$, the temperatures at $x_3=0$ and $x_3=d$ respectively, $S_0, S_0-\tilde{S}$ the concentration at $x_3=0$ and $x_3=d$ respectively, ρ_0 the density of the fluid at T_0 and α, α' the coefficients of volume and solute expansion respectively, then the fluid density is proportionate to θ and C such that

$$\rho=\rho_o(1-\alpha\theta+\alpha' C).$$

Thus the governing equations are

$$\begin{aligned}
&\dot{\upsilon}_i=-(P/\rho_o)_{,i}+\nu\nabla^2\upsilon_i-g(1-\alpha\theta+\alpha' C)\delta_{i3}+\frac{1}{\rho_o\mu}B_{i,k}B_k \\
&\quad -\frac{\nu}{k_1}\upsilon_i+2e_{ijk}\upsilon_j\Omega_k \ , \\
&\upsilon_{i,i}=0 \ , \\
&\dot{\theta}=\kappa\,\nabla^2\theta \ , \\
&\dot{C}=\kappa'\nabla^2 C \ , \\
&B_{i,i}=0 \ , \\
&J_i=e_{ijk}H_{k,j} \ , \\
&\frac{\partial B_i}{\partial t}=\upsilon_{i,j}-\upsilon_j B_{i,j}-\eta\, e_{ijk}\ J_{k,j}
\end{aligned} \tag{1}$$

where υ_i, H_i and P are velocity, magnetic field and pressure, ν, μ, k_1, κ, κ' and η are kinematic viscosity, magnetic permeability, permeability of porous medium, coefficient of thermal diffusivity, coefficient of solute diffusivity and electrical resistivity. Equations (1) have a steady state solution of the form

$$\begin{aligned}
&\upsilon_i=0\ ,\ J_i=0\ ,\ \theta=\theta(x_3)=T_o-\beta\,x_3\ ,\ P=P(x_3) \\
&C=C(x_3)=S_o-\beta' x_3,\ B_i=(0,0,B)\,B=constant
\end{aligned} \tag{2}$$

where $\beta=\tilde{T}/d$ is the uniform adverse temperature gradient and $\beta'=\tilde{S}/d$ is the uniform adverse concentration gradient. Suppose now that the variables in (2) have a small perturbation of the form

$$\begin{aligned}
&\upsilon_i=0+\varepsilon\,\hat{\upsilon}_i,\ J_i=0+\varepsilon\,\hat{J}_i,\theta=\theta+\varepsilon\,\hat{\theta}\ ,C=C+\varepsilon\,\hat{C}\ , \\
&P=P+\varepsilon\,\hat{P}\ ,\ B_i=(0,0,B)+\varepsilon\,\hat{b}_i
\end{aligned}$$

where $\hat{\upsilon}_i$, $\hat{J}_i,\hat{\theta},\hat{C},\hat{P}, b_i$ are the linear perturbations of velocity, current density, temperature, solute concentration, pressure and magnetic induction respectively. Following standard procedures the linear equations of (1) become

$$\begin{aligned}
&\frac{\partial\upsilon_i}{\partial t}=-P_{,i}+\nabla^2\upsilon_i+\sqrt{R_t}\ \theta\delta_{i3}-\sqrt{R_s}\ C\delta_{i3}-\frac{1}{N}\upsilon_i+b_{i,3} \\
&\qquad +\sqrt{T}\ e_{ijk}\ \upsilon_j\ \delta_{k3} \\
&\upsilon_{i,i}=0\ , \\
&P_r\frac{\partial\theta}{\partial t}+H\sqrt{R_t}\ \upsilon_3=\nabla^2\theta\ , \\
&P_r'\frac{\partial C}{\partial t}+H'\sqrt{R_s}\ \upsilon_3=\nabla^2 C\ , \\
&b_{i,i}=0\ , \\
&J_i=e_{ijk}\ b_{k,j} \\
&P_m\frac{\partial b_i}{\partial t}=Q\upsilon_{i,3}-e_{ijk}\ J_{k,j}\ .
\end{aligned} \tag{3}$$

Note that

$$R_t=\frac{\alpha\ g|\beta|}{\kappa\ \nu}\ d^4,\quad R_s=\frac{\alpha' g|\beta'|}{\kappa'\nu}\ d^4,\quad N=\frac{\kappa_1}{d^2},\quad P_r=\frac{\nu}{\kappa},$$

$$P_r'=\frac{\nu}{\kappa_1},\quad P_m=\frac{\mu\,\nu}{\eta},\quad Q=\frac{B^2d^2}{\rho_o\,\nu\,\eta},\quad T=\frac{4\Omega^2d^2}{\nu^2},$$

where R_t, R_s are the thermal and solute Rayleigh numbers, P_r, P_m are the viscous and magnetic Prandtle numbers, P_m is Schmidt number, Q is the Chandrasekhar number, T is the Taylor number and

$$H=\frac{-\beta'}{|\beta'|}=\begin{cases}1 & \text{when heating from above}\\ -1 & \text{when heating from below}\end{cases}$$

$$H'=\frac{-\beta'}{|\beta'|}=\begin{cases}1 & \text{when solute increases above}\\ -1 & \text{when solute increases below}\end{cases}$$

Boundary Conditions

The fluid layer is confined between the planes $x_3=0$ and $x_3=d$ and on these planes we need to specify mechanical, thermal and magnetic conditions. Mechanical conditions mean rigid or free boundaries. For rigid boundaries

$$\upsilon_{3,3}=0, \quad \xi_3=0 on \quad x_3=0,d \ ,$$

and for free boundaries

$$\upsilon_{3,33}=0, \quad \xi_{3,3} \quad on \quad x_3=0,d \ .$$

where ξ is third component of the vorticity. Thermal conditions mean an insulating or a conducting boundary. For a conducting boundary $\theta=\theta_{ext}$ and for an insulating boundary $\partial\theta/\partial x_3=0$, where θ_{ext} is the temperature outside the conducting boundary. Electromagnetic conditions assume either a stationary perfectly conducting electromagnetic boundary in which $b_3=0$ or a perfectly insulating electromagnetic boundary in which $(curl\, B)_3=0$. Solute conditions assume either an impermeable boundary in which $C_{,3}=0$ or a permeable boundary in which $C=C_{ext}$ where C_{ext} is the solute concentration in the exterior region.

The Eigenvalue Problem

Now we shall construct the related eigenvalue problem from equations (3) and the boundary conditions. We now apply a normal mode expansion of the form

$$\varphi=\varphi(x_3)\exp[i(nx_1+mx_2)+\sigma t]$$

where n,m are the wave numbers of the harmonic disturbance and σ is the growth rate. Thus equations (4) become

$$\begin{aligned}
&\sigma\,\xi=L\xi+DJ-\frac{1}{N}\xi+\sqrt{T}\ Dw,\\
&\sigma\,Lw=L^2w-a^2\sqrt{R_t}\ \theta+a^2\sqrt{R_s}\ C+L(Db)\\
&\qquad -\frac{1}{N}Lw-\sqrt{T}\ D\xi,\\
&\sigma\,P_m\ J=QD\xi+L\ J,\\
&\sigma\,P_m b=QDw+Lb,\\
&\sigma\,P_r\theta=L\theta-H\sqrt{R_t}\ w,\\
&\sigma\,P_r'C=LC-H'\sqrt{R_s}\ w.
\end{aligned} \tag{4}$$

where $a=\sqrt{n^2+m^2}$ is the wave number, $D=\partial/\partial x_3$ and $L=D^2-a^2$. We may eliminate J,b,θ,ξ,C from these equations to obtain a 14th order ordinary differential equation in w of the form

$$\begin{aligned}
&L(L-\sigma)(L-\sigma P_m)(L-\sigma P_r)(L-\sigma P_r')[(L-\sigma P_m)(L-\sigma-\frac{1}{N})\\
&-QD^2]w-a^2HR_t(L-\sigma P_m)(L-\sigma P_r')[(L-\sigma P_m)(L-\sigma-\frac{1}{N})\\
&-QD^2]w+a^2H'R_s(L-\sigma P_m)(L-\sigma P_r)[(L-\sigma P_m)(L-\sigma-\frac{1}{N})\\
&-QD^2]w-QL(L-\sigma P_r)(L-\sigma P_r')[(L-\sigma P_m)(L-\sigma-\frac{1}{N})\\
&-QD^2]D^2w-\frac{1}{N}L(L-\sigma P_m)(L-\sigma P_r)(L-\sigma P_r')[(L\\
&-\sigma P_m)(L-\sigma-\frac{1}{N})-QD^2]w+T(L-\sigma P_m)^2(L-\sigma P_r)(L\\
&-\sigma P_r')D^2w=0.
\end{aligned} \tag{5}$$

The Free Boundary Problem

Suppose that both boundaries are free and $w=A\sin(l\pi x_3)$, where A is a constant and l is an integer. Thus $Lw=-\lambda w$; $\lambda=l^2\pi^2+a^2$ and equation (5) becomes a sixth order polynomial in σ with real coefficients. Now we have four cases to consider.

Case 1. The porous layer is heated from above and the solute concentration decreases upwards

Here $H=1,\ H'=-1$. For stationary convection case we set $\sigma=0$ in equation (5) to obtain the thermal Rayleigh number

$$R_t=-[\frac{\lambda}{a^2C}\ (U^2+T\pi^2\lambda)+R_s]$$

where $U=(\lambda^2+\frac{\lambda}{N}+Q\ \pi^2)$ from which we deduce that no stationary convection stability. To study the possibility of overstability suppose that $\sigma_1\neq 0$ and we obtain from equation (5)

$$\begin{aligned}
R_{t1}=&-\frac{\gamma\ (\gamma+i\sigma_1)(\gamma+i\sigma_1P_r)}{(\gamma-1)}\ -\frac{(\gamma+i\sigma_1P_r)}{(\gamma+i\sigma_1P_r')}\ R_{s1}\\
&-\frac{\gamma\ (\gamma+i\sigma_1P_r)}{(\gamma-1)(\gamma+i\sigma_1P_m)}\ Q_1\ -\frac{\gamma\ (\gamma+i\sigma_1P_r)}{(\gamma-1)}\ \frac{1}{N_1}\\
&+\frac{(\gamma+i\sigma_1P_m)(\gamma+i\sigma_1P_r)}{(\gamma-1)[(\gamma+i\sigma_1P_m)(\gamma+i\sigma_1+\frac{1}{N_1})-Q_1]}\ T_1
\end{aligned}$$

where $\gamma=1+a_1$. To study the effect of solute Rayleigh number, magnetic field, permeability of porous

medium and rotation on the thermal Rayleigh number we have to discuss the nature of $\frac{dR_{t1}}{dR_{s1}}, \frac{dR_{t1}}{dQ_1}, \frac{dR_{t1}}{dN_1}, \frac{dR_{t1}}{dT_1}$.

In each of these derivatives we separate the real and imaginary parts, then solve the resulting equations to obtain :

$$\frac{dR_t}{dR_s} = -1,\ \frac{dR_t}{dQ_1} = -\frac{\gamma}{(\gamma-1)},\ \frac{dR_t}{dN_1} = -\frac{\gamma^2}{N_1{}^2(\gamma-1)},$$

$$\frac{dR_t}{dT_1} = -\frac{\gamma^2+\sigma_1{}^2 P_m P_r}{(\gamma-1)\left[\gamma(\gamma+1/N_1)+\sigma_1{}^2 P_m - Q_1\right]}$$

Thus all the parameters have a destabilizing effect. In fact rotation could have a stabilizing effect provided

$$Q_1 > \gamma(\gamma+\frac{1}{N_1}) + \sigma_1^2 P_m$$

Case 2. The porous layer is heated from above and the solute concentration increases upwards

Here $H=1,\ H'=1$. For stationary convection case we set $\sigma=0$ in equation (5) to obtain

$$R_t = -[\frac{\lambda}{a^2U}\ (U^2+T\pi^2\lambda)] + R_s. \tag{6}$$

Note that stationary convection stability is possible provided $R_s > \frac{\lambda}{a^2U}\ (U^2+T\pi^2\lambda).$

To study the effect of solute Rayleigh number, rotation, magnetic field and permeability of porous medium on R_t we deduce from (6) that

$$\frac{dR_t}{dR_s} = 1,\ \frac{dR_t}{dT} = -\frac{\lambda^2\pi^2}{a^2U},\ \frac{dR_t}{dQ} = -\left[\frac{\pi^2\lambda\, U^2 - \pi^4\lambda^2 T}{a^2U^2}\right],$$

$$\frac{dR_t}{dN} = -\left[\frac{\frac{\lambda^2}{N^2}(-U^2+T\,\pi^2\lambda)}{a^2U^2}\right].$$

Thus the solute Rayleigh number has a stabilizing effect, whereas rotation has a destabilizing effect. Magnetic field and permeability of porous medium have a destabilizing effect provided $\lambda\pi^2 T > U^2$. For overstability case we have

$$R_{t1} = -\frac{\gamma\ (\gamma+i\sigma_1)(\gamma+i\sigma_1 P_r)}{(\gamma-1)} + \frac{(\gamma+i\sigma_1 P_r)}{(\gamma+i\sigma_1 P_r')}\ R_{s1}$$
$$-\frac{\gamma\ (\gamma+i\sigma_1 P_r)}{(\gamma-1)(\gamma+i\sigma_1 P_m)}\ Q_1 - \frac{\gamma\ (\gamma+i\sigma_1 P_r)}{(\gamma-1)}\ \frac{1}{N_1}$$
$$+\frac{(\gamma+i\sigma_1 P_m)(\gamma+i\sigma_1 P_r)}{(\gamma-1)[(\gamma+i\sigma_1 P_m)(\gamma+i\sigma_1+\frac{1}{N_1})-Q_1]}\ T_1$$

From which we obtain

$$\frac{dR_{t1}}{dR_{s1}} = 1, \frac{dR_{t1}}{dQ_1} = -\frac{\gamma}{a_1}, \frac{dR_{t1}}{dN_1} = \frac{\gamma^2}{a_1\, N_1^2},$$

$$\frac{dR_{t1}}{dT_1} = \frac{\gamma^2+\sigma_1^2 P_m P_r}{a_1\left[\gamma(\gamma+\frac{1}{N_1})+\sigma_1^2 P_m - Q_1\right]}.$$

Thus solute Rayleigh number and permeability of porous medium have a stabilizing effect whereas magnetic field has a destabilizing effect. Moreover rotation has a stabilizing effect provided

$$Q_1 > \gamma(\gamma+\frac{1}{N_1}) + \sigma_1^2 P_m.$$

Case 3. The porous layer is heated from below and the solute concentration increases upwards

Here $H=-1,\ H'=1$. For stationary convection case we set $\sigma=0$ in equation (5) to obtain

$$R_t = \frac{\lambda}{a^2U}\ (U^2+T\pi^2\lambda) - R_s.$$

Clearly stationary convection stability is possible whenever $R_s < \frac{\lambda}{a^2U}\ (U^2+T\pi^2\lambda)$ and

$$\frac{dR_t}{dR_s} = -1,\ \frac{dR_t}{dT} = \frac{\lambda^2\pi^2}{a^2U},\ \frac{dR_t}{dQ} = \frac{\pi^2\lambda U^2 - \pi^4\lambda^2 T}{a^2U^2},$$

$$\frac{dR_t}{dN} = \frac{\frac{\lambda^2}{N^2}(-U^2+\pi^2\lambda\ T)}{a^2U^2}.$$

Thus solute Rayleigh number has a destabilizing effect, whereas rotation has a stabilizing effect. Magnetic field and permeability of porous medium have a destabilizing effect provided $\lambda\pi^2 T > U^2$ and permeability of porous medium has a stabilizing effect provided $\lambda\pi^2 T > U^2$. For overstability case

$$R_{t1} = \frac{\gamma\ (\gamma+i\sigma_1)(\gamma+i\sigma_1 P_r)}{(\gamma-1)} - \frac{(\gamma+i\sigma_1 P_r)}{(\gamma+i\sigma_1 P_r')}\ R_{s1}$$
$$+\frac{\gamma\ (\gamma+i\sigma_1 P_r)}{(\gamma-1)(\gamma+i\sigma_1 P_m)}\ Q_1 + \frac{\gamma\ (\gamma+i\sigma_1 P_r)}{(\gamma-1)}\ \frac{1}{N_1}$$
$$-\frac{(\gamma+i\sigma_1 P_m)(\gamma+i\sigma_1 P_r)}{(\gamma-1)[(\gamma+i\sigma_1 P_m)(\gamma+i\sigma_1+\frac{1}{N_1})-Q_1]}\ T_1$$

From which we obtain

$$\frac{dR_{t1}}{dR_{s1}} = -1, \frac{dR_{t1}}{dQ_1} = \frac{\gamma}{a_1}, \frac{dR_{t1}}{dN_1} = -\frac{\gamma^2}{a_1\, N_1^2},$$

$$\frac{dR_{t1}}{dT_1} = -\frac{\gamma^2+\sigma_1^2 P_m P_r}{a_1\left[\gamma(\gamma+\frac{1}{N_1})+\sigma_1^2 P_m - Q_1\right]}$$

i.e. solute Rayleigh number and permeability of porous medium have a destabilizing effect, whereas magnetic field has a stabilizing effect. Rotation has a stabilizing effect provided

$$Q_1 > \gamma(\gamma + \frac{1}{N_1}) + \sigma_1^2 P_{m1}\,.$$

Case 4. The porous layer is heated from below and the solute concentration decreases upwards

Here $H = -1,\ H' = -1$. For stationary convection case we set $\sigma = 0$ in equation (5) to obtain

$$R_t = \frac{\lambda}{a^2 U}\ (U^2 + T\pi^2\lambda) + R_s$$

and hence

$$\frac{dR_t}{dR_s} = 1,\quad \frac{dR_t}{dT} = \frac{\lambda^2\pi^2}{a^2 U},\quad \frac{dR_t}{dQ} = \frac{\pi^2\lambda\, U^2 - \pi^4\lambda^2\, T}{a^2 U^2}$$

$$\frac{dR_t}{dN} = \frac{\dfrac{\lambda^2}{N^2}(-U^2 + \pi^2\lambda\, T)}{a^2 U^2}.$$

Thus the solute Rayleigh number and rotation have a stabilizing effect on the system. However magnetic field has a stabilizing effect when $U^2 > \lambda\pi^2 T$ and the permeability of porous medium has a stabilizing effect when $\lambda\ \pi^2 T > U^2$. For the overstability case

$$R_{t1} = \frac{\gamma\ (\gamma + i\sigma_1)(\gamma + i\sigma_1 P_r)}{(\gamma - 1)} + \frac{(\gamma + i\sigma_1 P_r)}{(\gamma + i\sigma_1 P_r')}\ R_{s1} + \frac{\gamma\ (\gamma + i\sigma_1 P_r)}{(\gamma - 1)(\gamma + i\sigma_1 P_m)}\ Q_1 + \frac{\gamma\ (\gamma + i\sigma_1 P_r)}{(\gamma - 1)}\ \frac{1}{N_1} - \frac{(\gamma + i\sigma_1 P_m)(\gamma + i\sigma_1 P_r)}{(\gamma - 1)[(\gamma + i\sigma_1 P_m)(\gamma + i\sigma_1 + \frac{1}{N_1}) - Q_1]}\ T_1$$

from which we obtain

$$\frac{dR_{t1}}{dR_{s1}} = 1, \frac{dR_{t1}}{dQ_1} = \frac{\gamma}{a_1}, \frac{dR_{t1}}{dN_1} = -\frac{\gamma^2}{a_1 N_1^2}$$

$$\frac{dR_{t1}}{dT_1} = -\frac{\gamma^2 + \sigma_1^2 P_m P_r}{a_1\left[\gamma(\gamma + \dfrac{1}{N_1}) + \sigma_1^2 P_m - Q_1\right]}.$$

Thus the solute Rayleigh number and the magnetic field have a stabilizing effect whereas the permeability of porous medium has a destabilizing effect on the system. Rotation has a stabilizing effect on the system provided

$$Q > \gamma(\gamma + \frac{1}{N_1}) + \sigma_1^2 P_m\,.$$

Numerical Results and Discussion

The system of equations (5) together with the boundary conditions constitute an eigenvalue problem of order 14. This eigenvalue problem is solved using the Chebyshev Tau method when the fluid layer is heated from below and the solute concentration increases upwards which corresponds to case (3) in the previous section.

Numerical solutions are obtained for free and rigid boundaries for the following cases

(i) $DC = 0 \quad at \quad x_3 = 0, 1$.

(ii) $C = 0 \quad at \quad x_3 = 0$ and $DC = 0 \quad at \quad x_3 = 1$.

Stationary Convection

The relation between the magnetic parameter (Chandrasekhar number), *Q*, and the critical thermal Rayleigh number, R_t, is displayed in Fig. (1) when both boundaries are free and $T = 10^5$. As *Q* increases R_t increases for all values of *N* which indicates that magnetic field has a stabilizing effect. Moreover the values of R_t for case (i) of solute boundary conditions are less than the corresponding values of case (ii) of the conditions. In fact the difference between these values is very small when *N*=0.1 but when *N*=0.001 this difference is significant.

Figs. (1) shows also that as the permeability of the porous medium, *N*, decreases R_t increases which indicates that *N* has a destabilizing effect on the system. Moreover numerical results show that as *T* increases R_t increases for all values of *Q* and *N* which indicates that rotation has a stabilizing effect on the system. Similar effects are obtained when both boundaries are rigid. These effects are displayed in Fig. (2).

Overstability Case

The relation between the magnetic parameter, *Q*, and the critical thermal Rayleigh number, R_t, is displayed in Figs. (3) when both boundaries are free and $T = 10^5$ respectively. As *Q* increases R_t increases for all values of *T* when *N*=0.001 which indicates that magnetic field has a stabilizing effect. However as the permeability of porous medium increases unexpected results are obtained. In fact for small values of *Q*, as *Q* increases R_t decreases but for large values of *Q* as *Q* increases R_t increases as *Q* increases. These results coincide with those of Chandrasekhar (1981) where he considered the same problem in absence of porous medium.

Moreover for small values of *T* the values of R_t for case

(i) of solute boundary conditions are greater than the corresponding values of case (ii) of the conditions when N=0.1, however when N=0.001 the values of R_t for case (i) of solute boundary conditions are less. But for large values of T this conclusion is reversed.

Figs (3) shows also that as the permeability of the porous medium, N, decreases R_t increases which indicates that N has a destabilizing effect on the system. Moreover as T increases R_t increases for all values of Q and N which indicates that T has a stabilizing effect on the system.

We also note that the case of overstability do not appear unless $P_m > P_r$ and Q exceeds a certain critical value. This critical value is calculated when N = 0.1 and it is approximately equal to 1606 when T=0, 1568 when $T=10^3$, 1366 when $T=10^4$ and 380 when $T=10^6$. It appears that this critical value decreases as T increases. Similar effects are obtained when both boundaries are rigid. These effects are displayed in Fig. (4).

Fig. (5) shows the critical values of Q for T=50000 when N = 0.1 and N=0.001 for case (1) of solute boundary conditions when both boundaries are free.. The critical value of Q at which overstability is attained is calculated when N=0.001. This value is approximately equal to 2305 when T=0, 2350 when T=50000 and 2400 when $T=10^5$.

Conclusion

The thermohaline convection in a horizontal porous layer permeated by an incompressible solute fluid in the presence of magnetic field and rotation has been examined. Analytical solutions are obtained for different cases when both boundaries are free. Numerical solutions are obtained when both boundaries are free and rigid for the cases of stationary convection and overstability when the porous layer is heated from below and solute concentration increases upwards.

For the stationary convection and overstability cases, the presence of both magnetic field and rotation stabilize the system for different values of the permeability of porous medium. It appears also that as the fluid becomes less porous the fluid becomes more stable.

ACKNOWLEDGEMENTS

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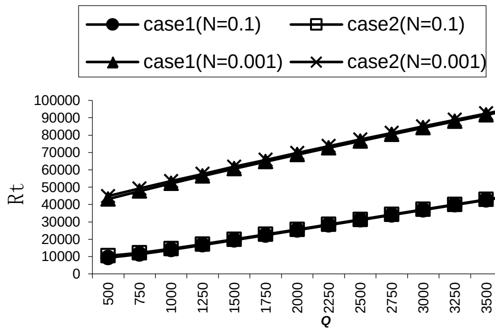

FIG (1): THE RELATION BETWEEN Q AND R_T FOR STATIONARY CASE WHEN BOTH BOUNDARIES ARE FREE, R_S = 5000 AND $T=10^5$

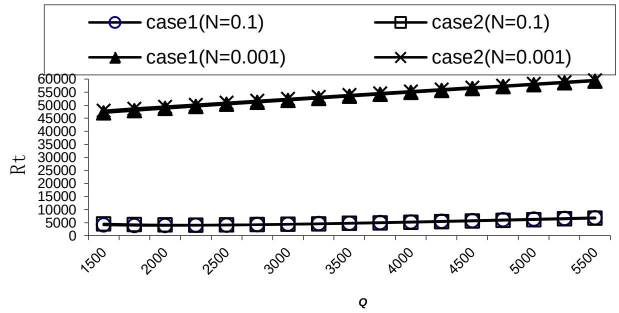

FIG (2) :THE RELATION BETWEEN Q AND RT FOR STATIONARY CASE WHEN BOTH BOUNDARIES ARE RIGID , RS = 5000 AND T=105

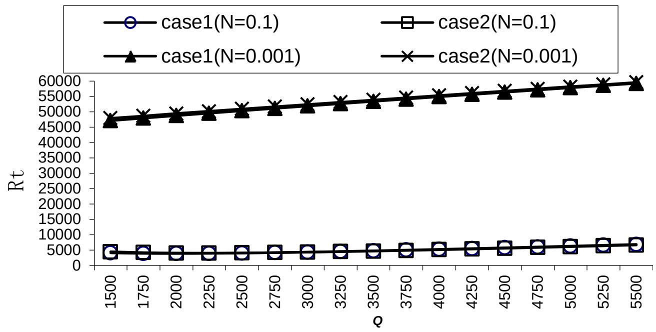

FIG (3): THE RELATION BETWEEN Q AND RT FOR OVERSTABILITY CASE WHEN BOTH BOUNDARIES ARE FREE, RS = 5000, T=105, PR =P'R =1 AND PM =6

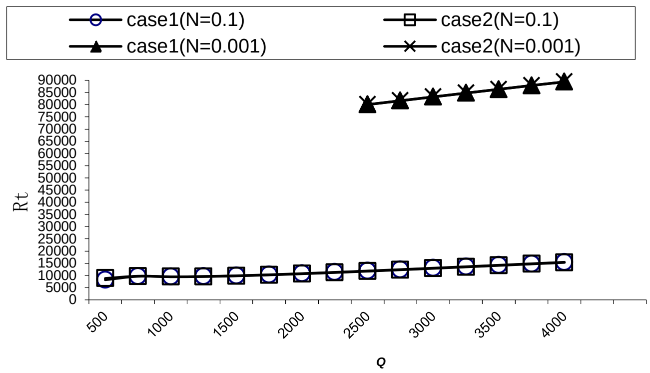

FIG (4): THE RELATION BETWEEN Q AND RT FOR OVERSTABILITY CASE WHEN BOTH BOUNDARIES ARE RIGID , RS = 5000 ,T= 105, PR =P'R =1 AND PM =6

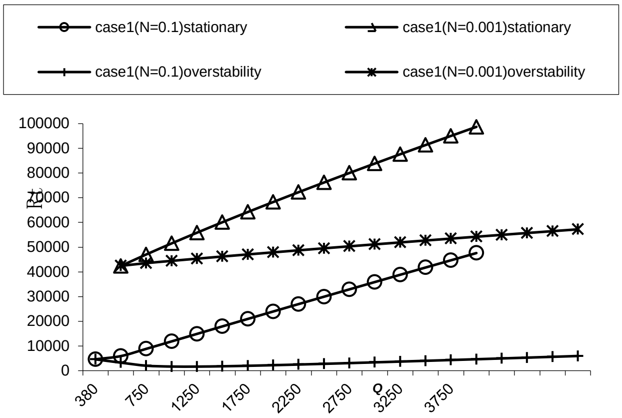

FIG (5): THE CRITICAL VALUES OF Q AT WHICH OVERSTABILITY CASE STARTS WHEN BOTH BOUNDARIES ARE FREE, RS = 5000 AND T=50000

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